

A LEVEL MATHEMATICS SUMMER WORK

Below you will find a selection of algebra and number-based tasks aimed at preparing you for the rigours of A Level Mathematics. Complete the questions ready to hand in on your first lesson in September. It should take you a few hours to complete. There are support notes included should you need any help.

We look forward to welcoming you to our Sixth form in September!

Indices and Surds

1 Evaluate each of these without using a calculator.

a $49^{\frac{1}{2}}$

b $27^{\frac{1}{3}}$

c 5^{-1}

d $64^{\frac{1}{3}}$

e $9^{\frac{3}{2}}$

f $16^{\frac{3}{4}}$

g $125^{\frac{2}{3}}$

h $\left(\frac{1}{2}\right)^3$

i $\left(\frac{1}{9}\right)^{-2}$

j $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

k $\left(\frac{9}{16}\right)^{-0.5}$

l $\left(\frac{27}{8}\right)^{\frac{2}{3}}$

2 Simplify these expressions fully without using a calculator.

a $\sqrt{8}$

b $\sqrt{75}$

c $2\sqrt{24}$

d $3\sqrt{48}$

e $\sqrt{20} + \sqrt{5}$

f $\sqrt{27} - \sqrt{12}$

g $5\sqrt{32} - 3\sqrt{8}$

h $\sqrt{50} + 3\sqrt{125}$

i $\sqrt{68} + 3\sqrt{17}$

j $3\sqrt{72} - \sqrt{32}$

k $4\sqrt{18} - 2\sqrt{3}$

l $6\sqrt{5} + \sqrt{50}$

3 Write each of these expressions in simplified index form.

a $x^3 \times x^7$

b $7x^5 \times 3x^6$

c $5x^4 \times 8x^7$

d $x^8 \div x^2$

e $8x^7 \div 2x^9$

f $3x^8 \div 12x^7$

g $(x^5)^7$

h $(x^2)^{-5}$

i $(3x^2)^4$

j $(6x^5)^2$

k $\sqrt{x^3}$

l $\sqrt[4]{x^5}$

m $\frac{5\sqrt{x}}{x}$

n $2x\sqrt{x}$

o $\frac{x^2}{3\sqrt{x}}$

p $x^3(x^5 - 1)$

q $x^3(\sqrt{x} + 2)$

r $\frac{x+2}{x^3}$

s $\frac{\sqrt{x}+3}{x}$

t $\frac{(3-x^3)}{\sqrt{x}}$

u $(\sqrt{x}+3)^2$

v $\frac{3+\sqrt{x}}{x^2}$

w $\frac{1-x}{2\sqrt{x}}$

x $\frac{\sqrt{x}+2}{3x^3}$

Solving & Rearranging Equations/Inequalities

1 Solve each of these linear equations.

a $3(2x+9)=7$

b $7-3x=12$

c $\frac{x+4}{5}=7$

d $2x+7=5x-6$

e $8x-3=2(3x+1)$

f $\frac{2x+9}{12}=x-1$

g $2(3x-7)=4x$

h $7-2x=3(4-5x)$

2 Solve each of these linear inequalities.

a $\frac{x}{2}+7 \geq 5$

b $3-4x < 15$

c $5(x-1) > 12+x$

d $\frac{x+1}{3} > 2$

e $8x-1 \leq 2x-5$

f $3(x+1) \geq \frac{x-3}{2}$

g $3(2x-5) < 1-x$

h $x-(3+2x) \geq 2(x+1)$

3 Rearrange each of these formulae to make x the subject.

a $2x+5=3A-1$

b $x+u=vx+3$

c $\frac{3x-1}{k}=2x$

d $5(x-3m)=2nx-4$

e $(1-3x)^2=t$

f $\frac{1}{x}=\frac{1}{p}+\frac{1}{q}$

g $\frac{1}{x^2+k}-6=4$

h $\sqrt{x+A}=2B$

Algebraic fractions

1 Simplify these fractions.

a $\frac{x(x-5)(x+2)}{x^3(x+2)}$ b $\frac{(x+3)^2}{x(x+3)}$ c $\frac{(x-4)}{2x(x-4)}$ d $\frac{x^2(x+5)}{x(x+5)^2}$

2 Simplify these fractions by first factorising the numerator and the denominator.

a $\frac{x^2-2x-8}{x^2+4x+4}$ b $\frac{x^2-10x+21}{x^2-x-6}$ c $\frac{x^2-3x-10}{x^2-10x+25}$ d $\frac{x^2+10x+24}{2x+8}$ e $\frac{x^2+6x}{x^2-36}$
f $\frac{3x^2+6x}{x^2-5x-14}$ g $\frac{5x^3+15x^2}{x^2+6x+9}$ h $\frac{x^2-64}{3x^2-24x}$ i $\frac{25-x^2}{45-4x-x^2}$ j $\frac{2x^2-x-28}{2x^3+7x^2}$

Lines

1 Find the gradient of the line through each pair of points.

a (3, 7) and (2, 8) b (5, 2) and (-4, -6) c (1.3, 4.7) and (2.6, -3.1)
d $\left(\frac{1}{2}, \frac{1}{3}\right)$ and $\left(\frac{3}{4}, \frac{2}{3}\right)$ e $(\sqrt{3}, 2)$ and $(2\sqrt{3}, 5)$ f $(3a, a)$ and $(a, 5a)$

2 Calculate the exact distance between each pair of points.

a (8, 4) and (1, 3) b (-3, 9) and (12, -7) c (5.9, 6.2) and (-8.1, 3.8)
d $\left(\frac{1}{5}, -\frac{1}{5}\right)$ and $\left(\frac{3}{5}, -\frac{4}{5}\right)$ e $(5, -3\sqrt{2})$ and $(2, \sqrt{2})$ f $(k, -3k)$ and $(2k, -6k)$

3 Find the coordinates of the midpoint of each pair of points.

a (3, 9) and (1, 7) b (2, -4) and (-3, -9) c (2.1, 3.5) and (6.3, -3.7)
d $\left(\frac{2}{3}, -\frac{1}{2}\right)$ and $\left(-\frac{5}{3}, -\frac{3}{2}\right)$ e $(6\sqrt{5}, 2\sqrt{5})$ and $(-\sqrt{5}, \sqrt{5})$ f $(m, 2n)$ and $(3m, -2n)$

4 Find the equation of the line through each pair of points.

a (2, 5) and (0, 6) b (1, -3) and (2, -5) c (4, 4) and (7, -7)
d (8, -2) and (4, -3) e (-3, -7) and (5, 9) f $(\sqrt{2}, -\sqrt{2})$ and $(3\sqrt{2}, 4\sqrt{2})$

5 Use algebra to find the point of intersection between each pair of lines.

a $y=8-3x, y=2-5x$ b $y=7x-4, y=3x-2$ c $y=2x+3, y=5-x$
d $y+5=3x, y=-5x+7$ e $y=\frac{1}{2}x+3, y=5-2x$ f $y=3(x+2), y=7-2x$

6 The line l_1 has equation $2x+6y=5$. The line l_2 is parallel to l_1 and passes through the point (1, -5). Find the equation of l_2 in the form $ax+by+c=0$ where a, b and c are integers.

7 Decide whether or not each line is parallel or perpendicular to the line $y=4x-1$

a $2x+8y=5$ b $20x+5y=2$ c $16x-4y=5$

Decide whether or not each line is parallel or perpendicular to the line $y=4-3x$

a $3x+6y=2$ b $5x-15y=7$ c $18x+6y+5=0$

8 The line l_1 has equation $4x+6y=3$. A second line, l_2 is perpendicular to l_1 and passes through the point (-1, 5). Find the equation of l_2 in the form $ax+by+c=0$ where a, b and c are integers.

Quadratics

1 Fully factorise each of these quadratics.

a $3x^2 + 5x$

b $8x^2 - 4x$

c $17x^2 + 34x$

d $18x^2 - 24x$

6 Use factorisation to find the roots of these quadratic equations.

a $21x^2 - 7x = 0$

b $x^2 - 36 = 0$

c $17x^2 + 34x = 0$

d $6x^2 + 13x + 5 = 0$

e $4x^2 - 49 = 0$

f $x^2 = 7x + 18$

g $x^2 - 7x + 6 = 0$

h $21x^2 = 2 - x$

7 Sketch each of these quadratic functions, labelling where they cross the x and y axes.

a $y = x(x - 3)$

b $y = -x(3x + 2)$

c $y = x(3 - x)$

d $y = (x + 2)(x - 2)$

e $y = (x + 4)^2$

f $y = 15x - 10x^2$

g $y = 49 - x^2$

h $y = -x^2 + 2x + 3$

i $y = x^2 - 4x + 4$

j $y = -x^2 + 14x - 49$

k $y = 3x^2 + 4x + 1$

l $y = -2x^2 + 11x - 12$

Completing the Square

1 Write each of these quadratic expressions in the form $p(x + q)^2 + r$

a $x^2 + 8x$

b $x^2 - 18x$

c $x^2 + 6x + 3$

d $x^2 + 12x - 5$

e $x^2 - 7x + 10$

f $x^2 + 5x + 9$

g $2x^2 + 8x + 4$

h $3x^2 + 18x - 6$

i $2x^2 - 10x + 3$

j $-x^2 + 12x - 1$

k $-x^2 + 9x - 3$

l $-2x^2 + 5x - 1$

2 Use completing the square to find the turning point of each of these curves and state whether it is a maximum or a minimum.

a $y = x^2 + 14x$

b $y = x^2 - 18x + 3$

c $y = x^2 - 9x$

d $y = -x^2 + 4x$

e $y = x^2 + 11x + 30$

f $y = -x^2 + 6x - 7$

g $y = 2x^2 + 16x - 5$

h $y = -3x^2 + 15x - 2$

Key Points & Examples to jog your memory if needed...

Indices & Surds

$$x^a \times x^b = x^{a+b} \quad x^a \div x^b = x^{a-b} \quad (x^a)^b = x^{ab}$$

Key point

The n th root of x is written $\sqrt[n]{x} = x^{\frac{1}{n}}$, and this can be raised to a power to give $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Key point

A power of -1 indicates a reciprocal, so $x^{-1} = \frac{1}{x}$ and, in general, $x^{-n} = \frac{1}{x^n}$

Key point

Example 1

Simplify these expressions. **a** $2x^3 \times 3x^5$ **b** $12x^7 \div 4x^6$ **c** $(3x^5)^3$

$$\begin{aligned} \mathbf{a} \quad 2x^3 \times 3x^5 &= 6x^{3+5} \\ &= 6x^8 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad 12x^7 \div 4x^6 &= \frac{12x^7}{4x^6} \\ &= 3x \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad (3x^5)^3 &= 3^3(x^5)^3 \\ &= 27x^{15} \end{aligned}$$

Multiply the coefficients together and use $x^a \times x^b = x^{a+b}$

Since $\frac{12}{4} = 3$ and $x^7 \div x^6 = x^{7-6} = x^1$ which we just write as x

Since $(x^a)^b = x^{ab}$

Both the 3 and the x^5 must be raised to the power 3

Example 2

Write these expressions in simplified index form.

a $\sqrt[3]{x}$ **b** $\frac{2}{x^3}$ **c** $\frac{2x}{\sqrt{x}}$

$$\mathbf{a} \quad \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\mathbf{b} \quad \frac{2}{x^3} = 2x^{-3}$$

$$\begin{aligned} \mathbf{c} \quad \frac{2x}{\sqrt{x}} &= \frac{2x}{x^{\frac{1}{2}}} \\ &= 2x^{1-\frac{1}{2}} \\ &= 2x^{\frac{1}{2}} \end{aligned}$$

Since $\sqrt{x} = x^{\frac{1}{2}}$

Subtract the powers, remembering that $x = x^1$

Example 3

Simplify these expressions without using a calculator.

a $\sqrt{18} + 5\sqrt{2}$ **b** $\frac{6}{\sqrt{3}}$ **c** $\frac{2}{1-\sqrt{5}}$

$$\begin{aligned} \text{a } \sqrt{18} &= \sqrt{9}\sqrt{2} \\ &= 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{Therefore } \sqrt{18} + 5\sqrt{2} &= 3\sqrt{2} + 5\sqrt{2} \\ &= 8\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{b } \frac{6}{\sqrt{3}} &= \frac{6\sqrt{3}}{\sqrt{3}\sqrt{3}} \\ &= \frac{6\sqrt{3}}{3} \\ &= 2\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{c } \frac{2}{1-\sqrt{5}} &= \frac{2(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})} \\ &= \frac{2(1+\sqrt{5})}{-4} \\ &= -\frac{1}{2}(1+\sqrt{5}) \end{aligned}$$

9 is a square-number factor of 18 so you can simplify $\sqrt{18}$

Collect like terms.

Rationalise the denominator by multiplying numerator and denominator by $\sqrt{3}$ Since $6 \div 3 = 2$ Rationalise the denominator by multiplying numerator and denominator by $1 + \sqrt{5}$

$$\begin{aligned} (1-\sqrt{5})(1+\sqrt{5}) &= 1 - \sqrt{5} + \sqrt{5} - 5 \\ &= 1 - 5 = -4 \end{aligned}$$

Solving & Rearranging Equations/Inequalities**Example 1**Solve the inequality $5(x-2) \leq 2x+1$

$$5x - 10 \leq 2x + 1$$

$$3x - 10 \leq 1$$

$$3x \leq 11$$

$$x \leq \frac{11}{3}$$

First expand the brackets.

Subtract $2x$ from both sides.

Add 10 to both sides.

Divide both sides by 3

Example 2Rearrange $Ax - 3 = \frac{x+B}{2}$ to make x the subject.

$$2Ax - 6 = x + B$$

$$2Ax - 6 - x = B$$

$$2Ax - x = B + 6$$

$$x(2A-1) = B+6$$

$$x = \frac{B+6}{2A-1}$$

Divide both sides by $(2A-1)$ to make x the subject.

Multiply both sides by 2

Subtract x from both sides.

Add 6 to both sides.

Factorise the side involving x

Algebraic Fractions

Example

Simplify the fractions **a** $\frac{x^2+5x+6}{2x^2+6x}$

b $\frac{2x^2-3x-9}{4x^2-9}$

a $x^2+5x+6=(x+2)(x+3)$

$2x^2+6x=2x(x+3)$

So $\frac{x^2+5x+6}{2x^2+6x} = \frac{(x+2)(x+3)}{2x(x+3)} = \frac{x+2}{2x}$

b $2x^2-3x-9=2x^2-6x+3x-9$

$=2x(x-3)+3(x-3)$

$= (2x+3)(x-3)$

$4x^2-9=(2x+3)(2x-3)$

So $\frac{2x^2-3x-9}{4x^2-9} = \frac{(2x+3)(x-3)}{(2x+3)(2x-3)} = \frac{x-3}{2x-3}$

Factorise numerator and denominator.

Cancel the common factor of $x+3$ from numerator and denominator.

Write $-3x$ as $-6x+3x$ since $3 \times (-6) = -18$ and $2 \times (-9) = -18$

Factorise in pairs.

Recognise the denominator as a difference of two squares.

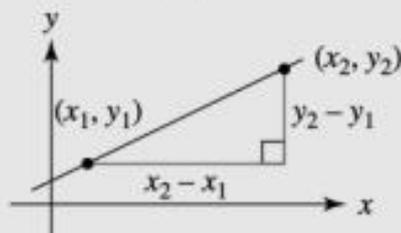
Cancel common factor $2x+3$ from numerator and denominator.

Lines

The gradient, m , of a line between two points (x_1, y_1)

and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$

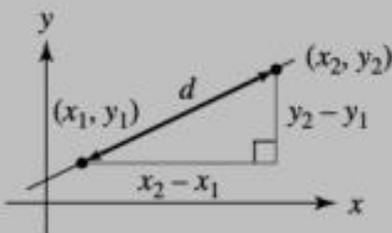
Key point



The length of the line segment, d , between two points

(x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Key point



The midpoint of the line segment from (x_1, y_1) to (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

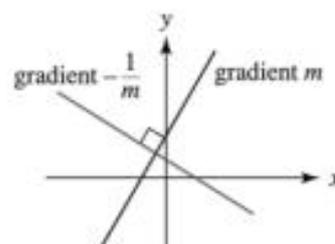
Key point

The equation of a straight line is $y = mx + c$ where m is the gradient and c is the y -intercept.

Key point

If the gradient of a line is m then the gradient of a perpendicular line is $-\frac{1}{m}$ since $m \times \left(-\frac{1}{m}\right) = -1$

Key point



Example 1

Work out the gradient and the y-intercept of each of these lines.

a $y = \frac{1}{2}x + 4$ **b** $y + x = 5$ **c** $-2x + 3y + 7 = 0$

a Gradient = $\frac{1}{2}$ and y-intercept = 4

b $y = 5 - x$

So gradient = -1 and y-intercept = 5

c $3y = -7 + 2x$

$$y = -\frac{7}{3} + \frac{2}{3}x$$

So gradient = $\frac{2}{3}$ and y-intercept = $-\frac{7}{3}$

Since $y = mx + c$ where m is the gradient and c is the y-intercept.

Rearrange to make y the subject.

Rearrange to make y the subject.

Example 2

Solve the simultaneous equations $5x - 4y = 17$, $3x + 8y = 5$

$15x + 40y = 25$ (1)

$15x - 12y = 51$ (2)

(1) - (2): $52y = -26$

$$y = -\frac{1}{2}$$

$5x - 4\left(-\frac{1}{2}\right) = 17$

$5x + 2 = 17$

$5x = 15$

$x = 3$

Multiply the second equation by 5

Multiply the first equation by 3

Subtract equation (2) from equation (1) to eliminate x

Solve this equation to find the value of x

Substitute $y = -\frac{1}{2}$ into one of the original equations.

Example 3

Decide whether or not each line is parallel or perpendicular to the line $y = 4x - 1$

a $2x + 8y = 5$ **b** $20x + 5y = 2$ **c** $16x - 4y = 5$

First note that the gradient of $y = 4x - 1$ is 4

a $2x + 8y = 5 \Rightarrow 8y = 5 - 2x$

$$\Rightarrow y = \frac{5}{8} - \frac{1}{4}x$$

$4x\left(-\frac{1}{4}\right) = -1$ so this line is perpendicular to $y = 4x - 1$

b $20x + 5y = 2 \Rightarrow 5y = 2 - 20x$

$$\Rightarrow y = \frac{2}{5} - 4x$$

The gradient is -4 so this line is neither parallel nor perpendicular to $y = 4x - 1$

c $16x - 4y = 5 \Rightarrow 4y = 16x - 5$

$$\Rightarrow y = 4x - \frac{5}{4}$$

The gradient is 4 so this line is parallel to $y = 4x - 1$

Rearrange to make y the subject.

The gradient is $-\frac{1}{4}$

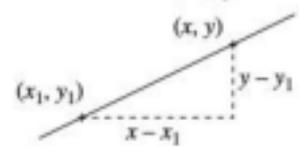
Since the product of the gradients is -1

Rearrange to make y the subject.

You can write the gradient of a line in terms of a known point on the line (x_1, y_1) , the general point (x, y) , and the gradient, m .

$$m = \frac{y - y_1}{x - x_1} \text{ or alternatively } y - y_1 = m(x - x_1)$$

$$\text{Gradient} = m = \frac{y - y_1}{x - x_1}$$



The equation of the line with gradient m through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$

Key point

Example 4

The line l_1 has equation $7x + 4y = 8$. The line l_2 is perpendicular to l_1 and passes through the point $(7, 3)$. Find the equation of l_2 in the form $ax + by + c = 0$ where a, b and c are integers.

$$l_1: 7x + 4y = 8 \Rightarrow 4y = -7x + 8$$

$$\Rightarrow y = -\frac{7}{4}x + 2$$

So the gradient of l_1 is $-\frac{7}{4}$ and the gradient of l_2 is $\frac{4}{7}$

So the equation of l_2 is $y - 3 = \frac{4}{7}(x - 7)$

$$\Rightarrow 7y - 21 = 4(x - 7)$$

$$\Rightarrow 7y - 21 = 4x - 28$$

$$\Rightarrow 4x - 7y - 7 = 0$$

Rearrange to the correct form.

Rearrange to make y the subject so you can see what the gradient is.

$$\text{Since } \left(-\frac{7}{4}\right) \times \frac{4}{7} = -1$$

Use $y - y_1 = m(x - x_1)$ to write the equation of l_2

Multiply both sides by 7 so that all coefficients are integers.

Quadratics

When factorising quadratics of the form $ax^2 + bx + c$ with $a \neq 1$, first split the bx term into two terms where the coefficients multiply to give the same value as $a \times c$

Example 1

Factorise each of these quadratics.

a $3x^2 + 11x + 6$

b $2x^2 - 9x + 10$

$$\mathbf{a} \quad 3x^2 + 11x + 6 = 3x^2 + 9x + 2x + 6$$

$$= 3x(x + 3) + 2(x + 3)$$

$$= (3x + 2)(x + 3)$$

$$\mathbf{b} \quad 2x^2 - 9x + 10 = 2x^2 - 4x - 5x + 10$$

$$= 2x(x - 2) - 5(x - 2)$$

$$= (2x - 5)(x - 2)$$

Split $11x$ into $9x + 2x$ since $9 \times 2 = 18$ and $3 \times 6 = 18$

Factorise the first pair of terms and the second pair of terms.

Split $9x$ into $-4x - 5x$ since $-4 \times -5 = 20$ and $2 \times 10 = 20$

Factorise the first pair of terms and the second pair of terms.

Example 2

Use factorisation to find the roots of these quadratic equations.

a $4x^2 + 12x = 0$ **b** $5x^2 = 21x - 4$

a $4x^2 + 12x = 4x(x+3)$

$4x(x+3) = 0 \Rightarrow 4x = 0$ or $x+3 = 0$

If $4x = 0$ then $x = 0$ and if $x+3 = 0$ then $x = -3$

b $5x^2 - 21x + 4 = 0$

$5x^2 - 21x + 4 = 5x^2 - 20x - x + 4$

$= 5x(x-4) - (x-4)$

$= (5x-1)(x-4)$

$(5x-1)(x-4) = 0 \Rightarrow 5x-1 = 0$ or $x-4 = 0$

If $5x-1 = 0$ then $x = \frac{1}{5}$ and if $x-4 = 0$ then $x = 4$

Factorise the quadratic.

One of the factors must be equal to zero.

Solve to find the roots.

Rearrange so you have a quadratic expression equal to zero.

Write $-21x = -x - 20x$ since $-20 \times -1 = 20$ and $5 \times 4 = 20$

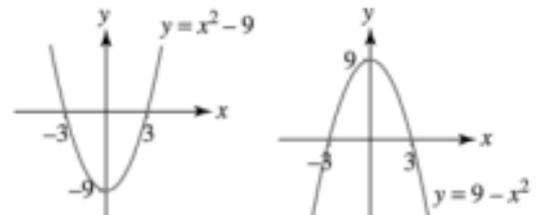
Factorise the quadratic.

The product is zero so one of the factors must be equal to zero.

Solve to find the roots.

A quadratic function has a **parabola** shaped curve.

When you sketch the graph of a quadratic function you must include the coordinates of the points where the curve crosses the x and y axes.



Example 3

Sketch these quadratic functions.

a $y = x^2 + x - 6$ **b** $y = -x^2 + 4x$

a When $x = 0$, $y = -6$

When $y = 0$, $x^2 + x - 6 = 0$

$x^2 + x - 6 = (x+3)(x-2)$

$(x+3)(x-2) = 0 \Rightarrow x = -3$ or $x = 2$

b When $x = 0$, $y = 0$

When $y = 0$, $-x^2 + 4x = 0$

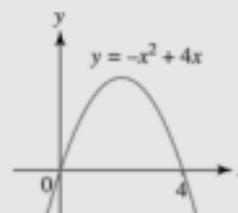
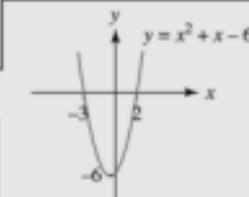
$-x^2 + 4x = -x(x-4)$

$-x(x-4) = 0 \Rightarrow x = 0$ or $x = 4$

Factorise to find the roots.

Find the y -intercept by letting $x = 0$

Find the x -intercept by letting $y = 0$



Find the y -intercept by letting $x = 0$

Find the x -intercept by letting $y = 0$

Factorise to find the roots.

Sketch the parabola and label the y -intercept of -6 and the x -intercepts of -3 and 2

Sketch the parabola, it will be this way up since the x^2 term in the quadratic is negative. Label the x and y intercepts.

The completed square form of $x^2 + bx + c$ is $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$

If you have an expression of the form $ax^2 + bx + c$ then first factor out the a , as shown in Example 1

Example 1

Write each of these quadratics in the form $p(x+q)^2 + r$ where p , q and r are constants to be found.

a $x^2 + 6x + 7$ **b** $-2x^2 + 12x$

$$\begin{aligned} \mathbf{a} \quad x^2 + 6x + 7 &= \left(x + \frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 7 \\ &= (x+3)^2 - 9 + 7 \\ &= (x+3)^2 - 2 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad -2x^2 + 12x &= -2[x^2 - 6x] \\ &= -2[(x-3)^2 - 9] \\ &= -2(x-3)^2 + 18 \end{aligned}$$

The constant term in the bracket will be half of the coefficient of x

First factor out the coefficient of x^2 then complete the square for the expression in the square brackets.

The turning point on the curve with equation $y = p(x+q)^2 + r$ has coordinates $(-q, r)$, this will be a minimum if p is positive and a maximum if p is negative.

Key point



Example 2

Find the coordinates of the turning point of the curve with equation $y = -x^2 + 5x - 2$

$$\begin{aligned} -x^2 + 5x - 2 &= -\left[x^2 - 5x + 2\right] \\ &= -\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 2\right] \\ &= -\left[\left(x - \frac{5}{2}\right)^2 - \frac{17}{4}\right] \\ &= -\left(x - \frac{5}{2}\right)^2 + \frac{17}{4} \end{aligned}$$

So the maximum point is at $\left(\frac{5}{2}, \frac{17}{4}\right)$

First factor out the -1 then complete the square for the expression in the square brackets.

The curve is at its highest point when the bracket is equal to zero: $x - \frac{5}{2} = 0 \Rightarrow x = \frac{5}{2}$